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ORIGINAL ARTICLE

The dual impact of antiretroviral therapy and sexual behaviour changes on HIV epidemiologic trends in Uganda: a modelling study

Leigh Anne Shafer,[1,2] Rebecca N Nsubuga,[2] Ruth Chapman,[3] Katie O'Brien,[3] Billy N Mayanja,[2] Richard G White[3]

▶ Additional material is published online only. To view please visit the journal online (http://dx.doi.org/10.1136/sextrans-2013-051219).

[1]Department of Internal Medicine, University of Manitoba, Winnipeg, Manitoba, Canada
[2]Medical Research Council Unit on AIDS/Uganda Virus Research Institute, Entebbe, Uganda
[3]London School of Hygiene and Tropical Medicine, London, UK

Correspondence to
Dr Leigh Anne Shafer, Department of Internal Medicine, University of Manitoba, GF335, 810 Sherbrook Avenue, Winnipeg, Manitoba, Canada R3E 3P5; leighanne.shafer@med.umanitoba.ca

Received 4 June 2013
Revised 29 January 2014
Accepted 2 February 2014
Published Online First 24 February 2014

To cite: Shafer LA, Nsubuga RN, Chapman R, *et al*. *Sex Transm Infect* 2014;**90**:423–429.

ABSTRACT

Objectives Antiretroviral therapy (ART) availability in a population may influence risky sexual behaviour. We examine the potential impact of ART on the HIV epidemic, incorporating evidence for the impact that ART may have on risky sexual behaviour.

Methods A mathematical model, parameterised using site-specific data from Uganda and worldwide literature review, was used to examine the likely impact of ART on HIV epidemiologic trends. We varied assumptions about rates of initiating ART, and changes in sexual partner turnover rates.

Results Modelling suggests that ART will reduce HIV incidence over 20 years, and increase prevalence. Even in the optimistic scenario of ART enrollment beginning after just five months of infection (in HIV stage 2), prevalence is estimated to rise from a baseline of 10.5% and 8.3% among women and men, respectively, to at least 12.1% and 10.2%, respectively. It will rise further if sexual disinhibition occurs or infectiousness while on ART is slightly higher (2% female to male, rather than 0.5%). The conditions required for ART to reduce prevalence over this period are likely too extreme to be achievable. For example, if ART enrolment begins in HIV stage 1 (within the first 5 months of infection), and if risky sexual behaviour does not increase, then 3 of our 11 top fitting results estimate a potential drop in HIV prevalence by 2025. If sexual risk taking rises, it will have a large additional impact on expected HIV prevalence. Prevalence will rise despite incidence falling, because ART extends life expectancy.

Conclusions HIV prevalence will rise. Even small increases in partner turnover rates will lead to an additional substantial increase in HIV prevalence. Policy makers are urged to continue HIV prevention activities, including promoting sex education, and to be prepared for a higher than previously suggested number of HIV infected people in need of treatment.

INTRODUCTION

Antiretroviral therapy (ART) use is now widespread in sub-Saharan Africa.[1] Recent evidence suggests that ART may reduce population-level incidence of HIV.[2] Modelling studies have also investigated the potential impact of ART.[3–12] These studies have reached varying conclusions. Treatment prolongs the lives of those infected, so some studies have suggested increased prevalence due to ART.[6 9 13] However, it has also been suggested that frequent testing and early ART enrolment could reduce HIV prevalence,[7 8 10] and may even eradicate the epidemic.[10 14] HIV-infected individuals on ART are less infectious than those not on ART.[15–17] The differing conclusions are largely dependent on assumptions, and may be particularly sensitive to assumptions about sexual behaviour.[12] ART may lead to sexual disinhibition as people feel that they may live a long life with HIV.

The overall impact that ART may have on the HIV epidemic sometimes incorporates the potential impact of ART on sexual behaviour.[6 10] However, the ranges of potential behaviour change are often not based on data, and do not include the impact that ART availability may have on behaviour among HIV uninfected people. Previous studies have provided contrasting results regarding the impact of ART on sexual behaviour.[18–20]

Previously, we examined self-reported evidence for changing sexual behaviour after the introduction of ART in a rural Ugandan cohort in 2004.[21] We found evidence that risky behaviour, particularly partner turnover rates, may rise among HIV uninfected people in response to the availability of ART.

In the Ugandan cohort mentioned above, HIV prevalence rose from 6.87% in 2004, the year that ART roll-out began, to 8.75% by 2012. However, in this cohort, most people do not begin ART treatment until HIV stage 3.

Here, through mathematical modelling, we assess the plausible impact of ART on future HIV prevalence and incidence under different scenarios of rate of ART enrolment. As some have postulated that ART may be a means of eradicating HIV,[10 14] our objective is to assess whether ART could conceivably reduce or eliminate the HIV epidemic, under extremely optimistic conditions. We examine impact in a range of scenarios with varying assumptions about the average time from HIV infection until ART enrolment, infectiousness while on ART, and sexual behaviour modification. Uniquely in this study, we incorporated evidence-driven potential sexual behaviour change due to ART among people infected with HIV, as well as due to the availability of ART among those uninfected with HIV.

METHODS

In 1989, the Medical Research Council established a general population cohort (GPC) in rural Uganda.[22–24] Participants are serotested annually for HIV. We present HIV prevalence estimates from

this cohort, following estimation methods described previously.[25 26] We fit our model to prevalence data through 2004, which is the last year in which the impact of ART on epidemiologic trends would not yet have been felt in this population.

Mathematical model

The compartmental mathematical model is stratified by age, sex and sexual activity group (figure 1). Details are online (see online supplementary appendix). Results are based on simulations with average durations in HIV stages 1–4 of 5 months, 7 years, 18 months and 10 months.

Upon ART enrolment in the model, infectiousness drops. At a rate of 0.5/year, those who have not yet died move into a compartment with mortality rate equivalent to background mortality. It is assumed that after surviving the initial ART period, life expectancy on ART is the same as that among HIV uninfected people.

Baseline scenario

The baseline scenario was fit to data from Uganda and literature reviews. We fit to empirically estimated HIV prevalence by gender among 15–54-year-olds from 1991 to 2004, before ART can have impacted prevalence. To fit, we ran 750 000 simulations with varying parameter values (see online supplementary table S1). Parameters include behaviour change before the introduction of ART, consistent with evidence suggesting that risky behaviour declined during the 1990s.[27] The parameter set with the highest goodness of fit between model and empirically estimated prevalence was considered the best fitting.

Table 1 ART Impact Scenarios

Scenario	Probability of HIV transmission while on ART (per partnership)	Earliest HIV stage of ART enrolment	Modelled change in sexual partner turnover?
1	0.5% F->M, 1.0% M->F	3 (rate of initiation varies)	No
2	2.0% F->M, 4.0% M->F	3 (rate of initiation varies)	No
3	0.5% F->M, 1.0% M->F	1 (rate of initiation varies)	No
4	2.0% F->M, 4.0% M->F	1 (rate of initiation varies)	No
5	0.5% F->M, 1.0% M->F	2 (rate of initiation=0.9/year)	Yes
6	2.0% F->M, 4.0% M->F	2 (rate of initiation=0.9/year)	Yes

ART, antiretroviral therapy.

ART impact scenarios

ART introduction was simulated in 2004. We examined six ART scenarios (table 1).

Within each of the first four scenarios, we varied the annual rate of ART enrolment once the respective HIV stage was reached, from 0.0 (baseline—no ART) to 1.0. In these scenarios, we assumed that partnership turnover rates did not change after ART introduction. We examined two scenarios in which ART enrolment may begin as early as HIV stage 1, and two in which ART enrolment may begin in HIV stage 3. We present the

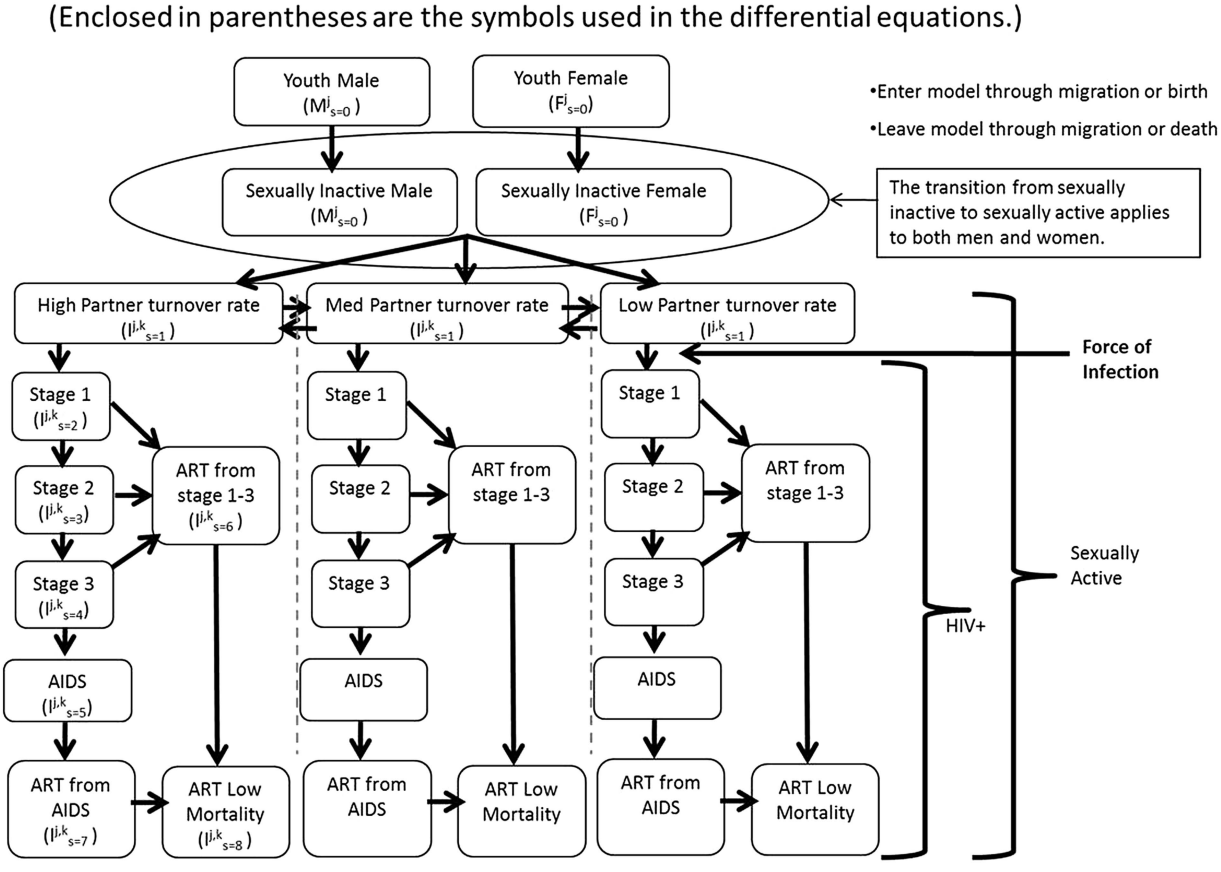

Figure 1 Compartmental Mathematical Model (Enclosed in parentheses are the symbols used in the differential equations.)

scenarios of ART enrolment beginning in stages 1 and 3 as these are boundaries. It may be unlikely to expect ART enrolment in stage 1 often enough to impact the epidemic, but we present this scenario as it represents the upper bound of the impact that ART could possibly have on the HIV epidemic.

When examining the impact of increasing partner turnover rates in response to ART availability, it was assumed that ART enrolment may begin as early as HIV stage 2, and that the enrolment rate was 0.9/year. The percent increase in partner turnover rate beginning in 2004 varied from 0.0% (no change) to 50.0%. HIV stage 2 represents a reasonable yet aggressive timing for ART enrolment.

When HIV transmission while on ART was 1% male to female (0.5% female to male), this represented a percent reduction in transmission probability in our best fitting scenario among men ranging from 85.1% to 96.6%, and among women ranging from 90.9% to 97.6%. The percent reduction depended on the stage from which ART was initiated, and the probability of transmission from the respective stage before ART enrolment.

Goodness-of-fit measures and sensitivity analysis

We assessed the sensitivity of results to parameter value set selection by summarising the results from multiple good fitting parameter sets: We found the best fitting parameter values using three goodness-of-fit (GoF) measures: a sum of squares, a χ^2,[28] and maximum likelihood.[29 30] We used three GoF measures because some readers may be more confident in results of one or another of the measures, but the three measures produced significantly overlapping best fits.[2] We compared results using the first through seventh best fitting parameter sets.

Of the 21 sets of parameter values, 10 were repeats. That is, five of the seven best fitting sets found when maximising the likelihood function were the same as those found when

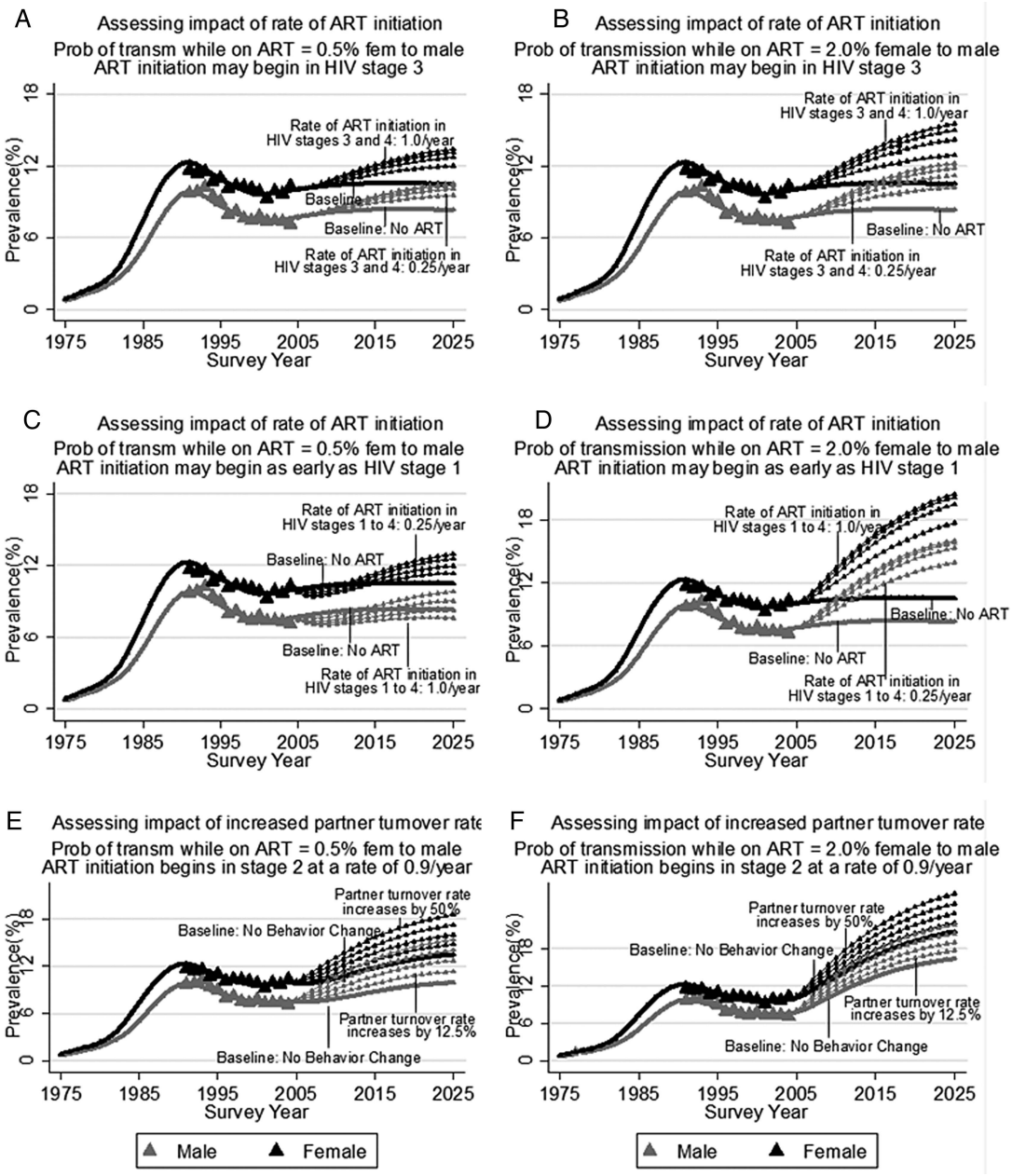

Figure 2 Sensitivity Analyses of Dual Impact of antiretroviral therapy (ART) and Potential Behavior Change on HIV Prevalence.

minimising the sum of squared differences, and five of those found under the χ^2 GoF had also been found under either the sum of squares or maximum likelihood GoF. We used the remaining 11 parameter sets to assess robustness of model results.

RESULTS

In the best fit, HIV transmission probability per partnership and stage is shown in online supplementary table S1.

Prevalence

Among the 11 best fitting simulations, we obtained model-estimated prevalence continuously from 1975 to 2025. The mean of these model estimates is presented in figure 2, for a range of different scenarios. Panels A and B correspond to scenario sets 1 and 2, described in methods. With no ART, HIV prevalence reaches 10.5% among women and 8.3% among men by 2025. In panel A, if the rate of ART enrolment beginning in stage 3 is 0.25/year, then predicted HIV prevalence rises to 12.0% among women and 9.5% among men by 2025. Panel B portrays a scenario set in which the drop in the probability of HIV transmission while on ART is less than that in panel A. In this case, under an otherwise similar scenario to that just described, prevalence rises to 12.9% among women and 10.2% among men by 2025. The increase in HIV prevalence is more pronounced in these scenarios if people begin ART at a faster rate. If ART enrolment beginning in HIV stage 3 is 1.0/year, then HIV prevalence may rise to 15.5% among women and 12.2% among men by 2025 (panel B).

In scenario set 3, people initiate ART in HIV stage 1, and there is no behaviour change in response to ART availability. This is the only one among the range of scenarios examined in which the average of our 11 best fitting prevalence estimates indicates that HIV prevalence may initially fall after ART introduction (figure 2C). By 10 years after ART is introduced, among men, HIV prevalence is higher if the rate of ART enrolment is less than 0.50/year than it would have been in the absence of ART, and among women it is higher regardless of ART enrolment rates. However, among men, if the rate of ART enrolment is greater than 0.50/year, and ART enrolment begins in HIV stage 1, then HIV prevalence may permanently decline, relative to what it is in the absence of ART. If the rate of ART enrolment is 1.0/year, HIV prevalence falls to 10.5% among women and 7.4% among men by 2015, before rising to 11.4% among women by 2025 and remaining level among men. Panel D (scenario set 4) portrays a similar scenario set, except that the drop in probability of HIV transmission while on ART is less. In this case, the impact of ART is to increase HIV prevalence dramatically, regardless of the rate at which people initiate ART.

We next examined the impact that ART may have on HIV prevalence if partner turnover rates increase. The scenarios depicted in figure 2E,F, are described as scenario sets 5 and 6 in methods. In panel E, a 12.5% increase in partner turnover rate results in rising HIV prevalence up to 20 years after ART is introduced. By 2025, prevalence reaches 17.7% among women and 13.9% among men, contrasting with 12.1% (women) and 10.2% (men) without behaviour change. If ART only reduces HIV transmission probability per partnership to 2% female to male (panel F), then prevalence is higher, but the relative impact of higher partner turnover rates is similar to what it is under the scenario in which ART reduces HIV transmission probability to 0.5% female to male.

As anticipated, we estimated increasing HIV prevalence in the presence of ART even when most of the HIV infected are on ART. In figure 2EF, the rate of initiating ART was 0.9/year, beginning in stage 2. In this case, the percent of HIV infected who are on ART reaches almost 90% by 20 years after the introduction of ART in the population.

Sensitivity analysis

Multiple sets of parameter values produced model estimates that fitted the empirically estimated prevalence. Without ART, the 11 sets of parameter values that most closely fit our data estimated similar HIV epidemics (figure 3).

By 2025, all 11 sets of parameter values estimated that ART would result in increased HIV prevalence, if ART enrolment begins in HIV stage 3 (table 2, scenario 1). The smallest increase in prevalence would occur if the probability of HIV transmission per partnership female to male was low at 0.5%, and the rate of ART enrolment was also low at 0.25/year. In this case, by 2025, the mean increase in prevalence was 14.6% (table 2, scenario 1). If either the rate at which people begin ART treatment or the probability of HIV transmission while on treatment rises, then all 11 parameter sets estimated an even greater rise in HIV prevalence than under the scenario just described.

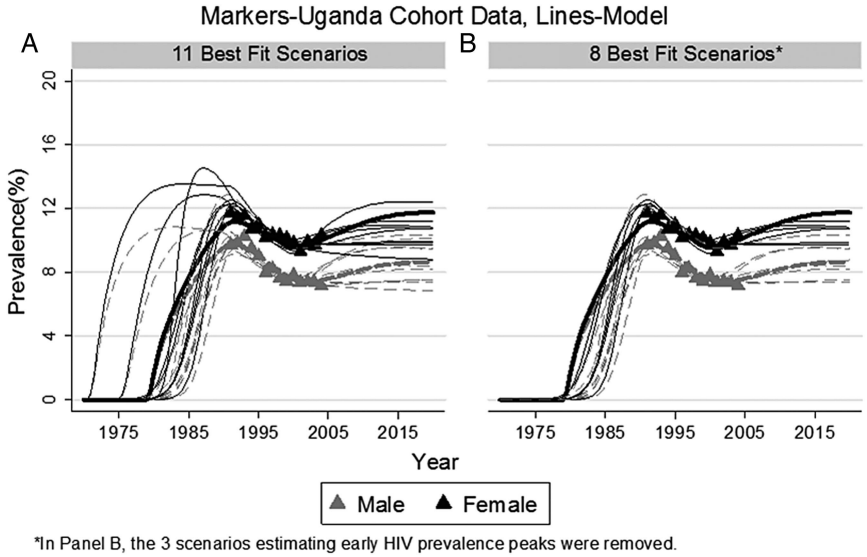

Figure 3 Prevalence by Gender–Age 15–54 Markers-Uganda Cohort Data, Lines-Model.

Table 2 Percent increase in HIV prevalence forecasted by 11 'best fit' parameter value sets, under scenarios depicted in figure 2 (% increase compared to scenario of no ART or no behaviour change)

		Modelled parameter value set											
Scenario	**Year**	**1**	**2**	**3***	**4**	**5**	**6**	**7**	**8**	**9**	**10***	**11***	**Mean (SD)**
Probability of transmission on ART=0.5% female to male. No ART enrolment in HIV stages 1 or 2 (figure 2A)													
Rate of ART init starting stage 3													
0.25/year	2015	1.3	11.2	0.4	−0.4	7.4	6.0	13.0	10.2	10.6	9.3	12.7	7.4 (4.9)
	2025	6.5	19.3	5.5	4.5	14.3	14.2	20.7	19.4	18.4	16.9	21.4	14.6 (6.3)
1.00/year	2015	1.8	23.4	−0.2	−3.7	14.4	11.5	28.6	21.8	23.2	20.2	27.9	15.4 (11.5)
	2025	9.9	39.0	7.9	1.6	26.8	27.2	43.7	39.5	38.2	33.9	45.0	28.4 (15.4)
Probability of transmission on ART=2.0% female to male. No ART enrolment in HIV stages 1 or 2 (figure 2B)													
Rate of ART init starting stage 3													
0.25/year	2015	6.6	14.8	4.9	2.5	15.5	14.8	17.8	17.6	16.2	16.8	15.0	13.0 (5.5)
	2025	16.7	24.1	13.9	10.4	25.8	27.1	27.0	31.5	26.2	31.2	24.7	23.5 (6.9)
1.00/year	2015	14.2	32.6	10.6	3.3	34.7	32.8	39.7	39.1	36.1	36.8	33.6	28.5 (12.7)
	2025	32.9	50.9	27.5	15.5	55.5	57.1	58.8	66.2	56.4	64.8	53.2	49.0 (16.3)
Probability of transmission on ART=0.5% female to male. ART enrolment begins in HIV stage 1 (figure 2C)													
Rate of ART init													
0.25/year	2015	−10.7	21.9	−10.7	−19.0	13.4	14.0	21.1	15.6	21.8	2.7	23.3	8.5 (15.4)
	2025	−4.1	44.7	−4.3	−21.5	30.1	37.5	42.3	39.4	44.2	14.4	49.2	24.7 (24.5)
1.00/year	2015	−30.9	17.1	−25.2	−33.6	6.7	10.9	17.2	7.7	24.5	−3.9	18.8	0.8 (21.2)
	2025	−41.1	35.5	−25.5	−46.8	12.0	27.4	32.2	21.6	47.5	1.2	43.7	9.8 (33.6)
Probability of transmission on ART=2.0% fem ale to male. ART enrolment begins in HIV stage 1 (figure 2D)													
Rate of ART init													
0.25/year	2015	16.0	44.2	12.2	−4.5	61.0	58.8	50.4	56.0	53.3	36.0	39.7	38.4 (21.6)
	2025	42.7	72.8	36.5	6.2	98.5	102.0	81.2	98.7	87.9	74.1	71.0	70.1 (30.1)
1.00/year	2015	17.2	69.9	14.7	−12.9	102.2	96.6	80.0	89.3	85.7	52.2	61.3	59.7 (37.9)
	2025	41.2	105.0	41.1	−10.6	151.5	153.9	119.2	143.1	129.8	95.5	100.4	97.3 (52.7)
Probability of transmission on ART=0.5% female to male. ART enrolment begins in stage 2 at rate 0.9/year (figure 2E)													
Change partner turnover rate													
Increase by 12.5%	2015	10.3	6.4	7.1	9.6	10.9	8.8	7.5	8.5	6.2	6.6	6.0	8.0 (1.7)
	2025	14.5	6.3	10.0	16.2	14.7	10.4	8.0	9.9	6.5	9.2	5.5	10.1 (3.6)
Increase by 50%	2015	43.2	19.9	32.9	65.6	40.9	34.4	25.0	31.0	24.0	30.9	20.6	33.5 (13.1)
	2025	58.7	19.2	45.7	110.0	48.0	36.1	25.4	33.9	24.1	43.2	18.6	42.1 (25.9)
Probability of transmission on ART=2.0% female to male. ART enrolment begins in stage 2 at rate 0.9/year (figure 2F)													
Change partner turnover rate													
Increase by 12.5%	2015	9.8	3.1	8.0	12.0	6.1	6.0	3.7	5.2	5.0	8.4	3.5	6.4 (2.8)
	2025	12.0	2.9	9.5	18.7	6.0	6.2	3.9	5.1	4.9	10.5	3.6	7.6 (4.7)
Increase by 50%	2015	41.6	11.6	35.1	71.1	24.8	24.8	14.4	20.6	20.7	38.7	12.9	28.8 (17.4)
	2025	47.8	10.8	39.2	101.2	23.4	25.1	15.3	20.2	19.4	47.8	13.1	33.0 (26.1)

*Though parameter value sets 3, 10, and 11 provided good model fits to empirically estimated prevalence, we believe they are less likely than the others because they estimated the HIV epidemic peaking too early (see figure 3).
ART, antiretroviral therapy.

In the extreme scenario in which ART was initiated beginning during HIV stage 1, HIV transmission probability on ART female to male was 0.5%, and there was no behaviour change, the estimated impacts from the 11 parameter sets were inconsistent. Three of the sets suggested a drop in HIV prevalence by 2025, while eight suggested an increase. Across all parameter sets, the mean change in HIV prevalence over the scenario of no ART was a 24.7% rise assuming that the rate of ART enrolment was 0.25/year and a 9.8% rise if the rate of ART enrolment was 1.00/year (table 2, scenario 3). If the probability of transmission while on ART was 2% female to male, all but one of the modelled parameter sets indicated that ART would result in rising HIV prevalence (table 2, scenario 4).

Not surprisingly, all sets of parameter values suggested that an increase in partner turnover rate would result in rising HIV prevalence. For example, if the probability of HIV transmission while on ART was just 0.5% female to male, and ART enrolment began in stage 2 at a very high rate of 0.9/year, then the mean increase in prevalence by 2025 caused by a 12.5% increase in partner turnover rate was 10.1% (table 2, scenario 5).

Incidence

Without behaviour change, ART reduces HIV incidence (figure 4). This assumes a high rate of ART enrolment beginning in HIV stage 2, and that ART lowers the probability of HIV infection per partnership to 0.5% female to male and 1% male to female. Without behaviour change, our model suggests that ART would lower incidence from 14.7 to 8.9 per 1000 person-years observation (pyo) among men, and from 19.3 to 13.7 among women. If average partner turnover rate increases by 12.5%, then our results indicate that the dual impact of ART and behaviour change would lower incidence to 10.3 per 1000 pyo among men, and to 15.3 among women. If partner turnover rate increases by 50%, then incidence would initially rise, but would then stabilise at approximately the same values as without ART.

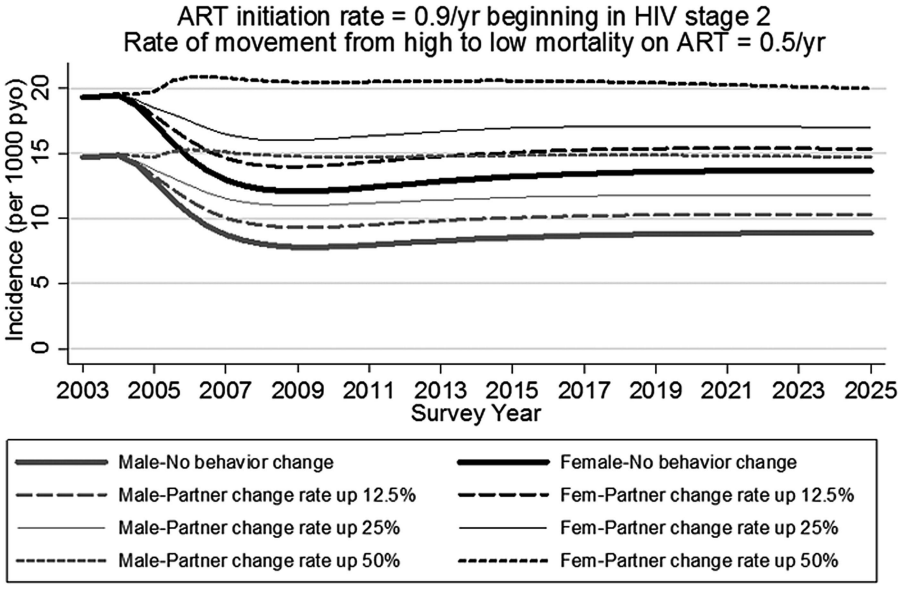

Figure 4 Incidence among age 15–54 antiretroviral therapy (ART) initiation rate=0.9/year beginning in HIV stage 2. Rate of movement from high to low mortality on ART=0.5/year.

In the figure 4 scenario, with no behaviour change, 18.4% of incidence by 2025 comes from people in stage 1, 6.6% from people in stage 2, and 73.4% from people on ART. Just 1.6% of incidence comes from people in stages 3 and 4. By contrast, in the same scenario but in the year prior to ART introduction, 36.2% of incidence comes from people in stage 1, 40.3% from stage 2, 16.5% from stage 3 and 7.0% from stage 4. The lower percent of incidence in stage 4 despite high infectiousness results from: a reduction in partner turnover rate in stage 4 (see online supplementary table S1), and mortality before stage 4 such that some people never reach this stage.

DISCUSSION

Epidemiologic forecast

Recent evidence from clinical trials have shown the positive impact that ART may have on reducing new infections (incidence).[2] Our modelling work supports this evidence by also showing a reduction in incidence, unless partner turnover rates increase greatly (>50%). However, although incidence falls, results suggested that HIV prevalence will be higher with ART than without. Among the scenarios examined, HIV prevalence may decrease among men only in an extreme case which may not be achievable in practice. That is, ART enrolment would need to begin while still in HIV stage 1, the rate of ART enrolment would need to be greater than 0.50/year, sexual partner turnover would not change, and the probability of transmission while on ART would be just 0.5% per partnership from female to male (representing >95% reduction in transmission compared to no ART). Even in this case, 8 of the 11 sets of parameter values that we assessed suggested that HIV prevalence would be higher in the presence of ART than in its absence.

Despite the short duration in the initial HIV stage of infection, our model estimates that in the absence of ART about 36% of incidence comes from partnerships formed in which the HIV-infected partner was in the highly infectious stage 1. Initiating ART during this stage, if feasible, would therefore have the greatest impact on the epidemic.

Previous studies have suggested a range of potential outcomes to rolling out ART, from increasing[6 9 13] to decreasing HIV prevalence.[7 8 10 14] Although our work supports this range of potential outcomes to rolling out ART, in all but the most extreme scenarios our results consistently suggest that HIV prevalence will rise as a result of ART.

Results from this study come from a model that was parameterised and fit using data from Uganda. We expect that our results provide a good estimate of the impact of ART and corresponding sexual behaviour change on HIV epidemiologic trends in countries that are similar to Uganda in terms of HIV prevalence trends pre-ART and HIV modes of transmission (mainly heterosexual sex).

Limitations

Results of any modelling study may be influenced by model assumptions. We have described our model assumptions in detail in the online supplementary appendix. One model assumption was that of no HIV/AIDS-related mortality after the initial period on ART. This is likely overly optimistic as non-perfect ART adherence speeds HIV progression and a higher mortality rate. If we had modelled a slightly higher mortality rate, even among those on ART, then estimated HIV prevalence after ART is introduced would be lower, as infected people would die at a higher rate. Also, we used a deterministic mathematical model. As such, stochastic events that may influence the plausible range of the impact of ART on HIV prevalence and incidence were not modelled.

In our best fitting model simulation, 36.2% of HIV incidence prior to ART introduction came from people in HIV stage 1. While this is similar to that estimated elsewhere,[31 32] it is higher than that estimated by some.[33] If more HIV incidence is attributable to higher HIV stages than we estimated, then ART enrolment in later stages would have a greater impact on incidence and subsequent prevalence.

CONCLUSION

It is unlikely that most HIV infections could be identified during the high viraemic stage 1 of infection. If infections are identified at a rate of 0.9/year beginning in stage 2, and ART is initiated immediately after detection, our results indicate that HIV incidence would fall as a result of ART. However, this comes at a cost. Our results still indicate rising prevalence as those infected live longer. The combined effect of increased risky sexual behaviour and longer life expectancy while infected would increase HIV prevalence dramatically. Care providers should be prepared

for a higher than previously suggested[11] number of people who are likely to need treatment.

Key messages

- Although HIV incidence will fall, HIV prevalence will rise as a result of antiretroviral therapy.
- If the possible increase in risky sexual behaviour is not controlled, HIV prevalence will rise even further.
- Policy makers must prepare to provide care for the increased population living with HIV.
- Policy makers are urged to continue HIV prevention activities, including promoting sex education.

Handling editor Jackie A Cassell

Contributors LAS, RNN, RGW, KO and RC developed the model. LAS fit the model and wrote the manuscript. LAS, RNN and RC conducted literature review. BNM directed the MRC cohort from which data was provided. All authors contributed suggestions to the manuscript.

Funding This work was supported by the Medical Research Council-UK, grant number G0501499. RGW is funded by the Medical Research Council (UK) (MR/J005088/1 and G0802414), the Bill and Melinda Gates Foundation (TB Modelling and Analysis Consortium: Grants 21675 / OPP1084276 and Consortium to Respond Effectively to the AIDS/TB Epidemic: 19790.01), and CDC/PEPFAR via the Aurum Institute (U2GPS0008111). The funders had no involvement in the design, collection, analysis or interpretation of the data, in writing the report or in the decision to submit.

Competing interests None.

Ethics approval This study was approved by the Science and Ethics Committee of the Uganda Virus Research Institute.

Provenance and peer review Not commissioned; externally peer reviewed.

Data sharing statement Data used to inform our mathematical model came from a 21-year cohort in rural Uganda. This is a sero-behavioral general population cohort, comprising 25 villages. The Medical Research Council in Uganda has data on numerous sexual behaviour indicators, collected annually from each member of the cohort, as well as annual HIV test results.

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Appendix I:
HIV Evolution Model
Description and Differential Equations

December, 2013

Contents

1 Model Description

The model was written using the Berkeley Madonna modelling software.

For infection of HIV, we consider the following populations:

$$M_s^{j,k}, \qquad F_s^{j,k} \tag{1}$$

denoting the male (M) and female (F) populations by designation according to the subscript state and superscript groups. The subscript denotes the HIV state s. The superscripts denote groups: age group j, and sexual activity group k. To distiguish between sexually active and sexually inactive populations we will use the HIV state $s = 0$ for sexually inactive people. We emphasize that the **superscripts denote the group** while **subscript denotes the state** within a given group.

1.1 Burn In

To allow the demographic composition of the population to reach current empirical age/sex demography, we begin each simulation with a 100-year burn-in. That is, we begin the simulation in 1870, and do not obtain model estimated results until 1970 at the earliest. We introduce HIV infection in the population by spontaneously infecting 0.0001% of sexually active males aged 20-29 in a user defined year between 1970 and 1980.

1.2 Age Group Parameters

The age group, denoted by subscript j, ranges from 0 (the lowest age group) to $J - 1$ the highest age group where there are J total age groups. An age group is defined by the start and end ages of the group. Although age groups may be supplied by the user with any range, our modelling work generally groups ages into 1-year intervals, up to age 70.

1.3 Sexual Activity Group Parameters

Sexual activity group k ranges from 0 to $K - 1$ where there are K total activity groups. Sexual activity is defined by desired sexual partner turnover rate and desired age mixing.

1.4 Age/Sexual Activity Input Parameters

Each doublet (j, k) defines a population (either male or female) existing in age group j, and sexual activity group k. For each such group we allow the following parameters:

1.4.1 Birth rate

We let $b^{(j)}$ be the number of births per unit time per female in age group j. We let b_m and b_f be the fraction of births that are males and females respectively. We enforce the constraint that $b_m + b_f = 1$. The rate of male or female births per unit time per female in a given age

group are then the products of these fractions and the corresponding rate $b^{(j)}$. Finally we assume that the user provides as input vectors that indicates what fraction of all births that end up in sexual activity group k upon reaching sexual debut. We denote these vectors as:

$$b_{m,k} \quad and \quad b_{f,k} \tag{2}$$

where we permit different vectors for males and females. *All of the above birth rate parameters are supplied as user input.*

1.4.2 Background Mortality rate

We let $\mu_m^{(j)}$ and $\mu_f^{(j)}$ denote the background male and female mortality rates (exclusive of HIV related mortality) for age group j. The units of these rates are deaths per unit time per person (as deaths are people these units are just frequency). *These mortality rates are supplied as user input.*

1.4.3 Age of sexual debut and becoming sexually active

We let $\vartheta^{(m,j)}$ and $\vartheta^{(f,j)}$ denote the rate of becoming sexually active per unit time per male or female, respectively, at a specific age j. Then, the earliest potential age of sexual debut, denoted $d^{i,j}$ where i is one of m or f for males or females, can be computed as the minimum j such that $\vartheta^{(i,j)}$ is non-zero. *The parameters $\vartheta^{i,j}$ are input by the user.*

1.4.4 Desired partnership turnover rate

We let $\Delta_s^{(m,j,k)}$ and $\Delta_s^{(f,j,k)}$ be the desired partner turnover rate for age group j and sexual activity group k for males and females respectively in HIV state s. Note that these values are identically zero for $s = 0$, the sexually inactive and HIV uninfected state. These rates are the total number of desired parters of the opposite sex independent of the partner's age group, sexual activity group, or HIV state. *These rates are specified by the user.*

1.4.5 Desired partner mixing parameters

We denote $m_{j'}^{(m,j,k)}$ and $m_{j'}^{(f,j,k)}$ as the percentage of partners that males and females in age group j and activity group k respectively want from age group j' subject to the following constraint:

$$\sum_{j'=0}^{J-1} m_{j'}^{(m,j,k)} = \sum_{j'=0}^{J-1} m_{j'}^{(f,j,k)} = 1 \tag{3}$$

For example, the desired partnership turnover rate can be specified with a target partner age j' such that:

$$\begin{aligned} \Delta_{s,j'}^{(m,j,k)} &= \Delta_s^{(m,j,k)} m_{j'}^{(m,j,k)} \\ \Delta_{s,j'}^{(f,j,k)} &= \Delta_s^{(f,j,k)} m_{j'}^{(f,j,k)} \end{aligned} \tag{4}$$

In general, the mixing matrix is supplied as a generic mixing range and a desired fraction of partners that are older and younger than the age group j. The mixing matrix conforms to

the fact that for certain age groups j, the corresponding age group j' within the mixing range may not contain a sexually active population. In this case, the fractional part of partners in older or younger age groups is adjusted accordingly. This may happen when people at the youngest age at sexual debut desire a fraction of partners to be younger than themselves, or when those at or near the oldest age in the model (supplied by the user but, e.g., age 70) desire a fraction of partners to be older than themselves.

1.4.6 Cross generational mixing

Notice that the age mixing matrix has both gender and sexual activity group dimensions. Age mixing may vary by sexual activity group. In our current work, some women are in sexual activity groups in which they are willing to have much older partners (cross-generational mixing with partners up to 30 years older than themselves). In the current work, this corresponds to sexual activity groups $k = 3$ to $k = 5$, in which women desire the same number of partners as women in groups $k = 0$ to $k = 2$, respectively, but with a different age mixing range. Women in these sexual activity groups desire 100% of their partners to be 1 to 30 years older than themselves. All men in the model may have some cross-generational mixing, but not all women, and the proportion of women who are willing to mix cross-generationally is supplied by the user as a model input.

1.4.7 Sexual Activity migration

We let the quantity $B_{k,k'}^{(m,j)}$ and $B_{k,k'}^{(f,j)}$ denote rates of migrating from sexual activity (Behaviour) group k to sexual activity group k' for age group j for males and females respectively. Though not dictated by the model structure, a common constraint in our work is that sexual activity migration does not directly occur between two non-adjoining sexual activity groups. For example, $B_{1,3}^{(m,j)} = 0$. *These values are supplied by the user.*

1.5 Population States

Having defined a population group (recall this is the superscripts) and all of the parameters related (primarily) to the group, we can now discuss the different **states** that exist in this population (recall that a state is given by the subscript sx). Recall that our populations are denoted by:

$$M_s^{j,k}, \qquad F_s^{j,k} \tag{5}$$

Here we focus on the subscript s which denotes the possible HIV states of the total population (j, k).

1.5.1 HIV state

The subscript s denotes the HIV state of the population. We distinguish between s=0, the HIV uninfected state among sexually inactive groups and s=1, the HIV uninfected state

among sexually active groups. In addition, there are four HIV stages while not on anti-retroviral therapy (ART), the last stage representing AIDS. The four HIV stages are denoted by s=2 to s=5. There are also three HIV states related to HIV infected people on ART, s=6 to s=8. These represent those transiting to ART from HIV stages 1-3 (not yet AIDS), those transiting to ART from HIV stage 4 (AIDS), and those who have been on ART long enough that their HIV-related mortality rate has disappeared so that they only die from background (non HIV-related) mortality.

The rate of transition from one state to another is provided as user input and may include a rate of 0 (for example, the user will not wish to allow those in HIV stage 3, represented by $s = 4$, to transit to HIV stage 2, represented by $s = 3$).

1.6 HIV State Input Parameters

The following input parameters are defined for a given state.

1.6.1 HIV stage and ART transition rates

For any two HIV infected states including those on ART treatment, we let $\gamma_{s,s'}^{(i,j)}$ denote the rate of transition from state s to state s', for gender i and age j. These may equal 0. There are two special cases, both involving HIV uninfected populations. As a reminder, the rate of transition between $s = 0$ (sexually inactive and HIV uninfected) to $s = 1$ (sexually active and HIV uninfected) is determined by $\vartheta^{(m,j)}$ and $\vartheta^{(f,j)}$ for males and females respectively. The second special case involves the rate of transition from $s = 1$ to $s = 2$. This is the force of infection.

1.6.2 HIV/AIDS-related Mortality rate

We let $\mu_s^{(j)}$ denote the HIV stage-dependent mortality. This represents the excess mortality that individuals in HIV stage s incur, over and above background mortality. These mortality rates are independent of gender and sexual activity group. These mortality rates will, by definition, equal 0 for stages $s = 0$ (sexually inactive and HIV uninfected) and $s = 1$ (sexually active but HIV uninfected). The units of these rates are deaths per unit time per person (as deaths are people these units are just frequency). *These mortality rates are supplied as user input.*

1.6.3 HIV infection rate (force of infection)

The force of infection HIV is the heart of the algorithm and will be discussed in a section on its own later. For now, we simply denote the force of infection as $\lambda^{(m,j,k)}$ and $\lambda^{(f,j,k)}$ to denote the force of infection for males and females respectively. We let the force of infection depend on all population group parameters. Before giving the force of infection equations, we now provide the differential equations for the rate of change of population. We will, however,

have as inputs to the model the probability of HIV infection per sex act with an infected partner, which we denote $\phi_{s'}^{(m,j,k)}$ for men and $\phi_{s'}^{(f,j,k)}$ for women. Note that these probabilities depend on the HIV state s' of the partner. We will also supply the number of sex acts per partnership $\psi^{(m,j,k)}$, $\psi^{(f,j,k)}$ which we assume can vary with the index person's gender, age and sexual activity group.

The model was designed such that the user can decide whether to provide probability of HIV transmission per partnership, or per sex act. If the user wishes to base the model on HIV transmission probability per partnership, then the number of sex acts per partnership may be entered as 1, and the probability of transmission per sex act may be used to represent the probability of transmission per partnership.

2 Model Equations

Let I represent either a male or female population. The model equations are then:

$$\begin{aligned}
\frac{dI_s^{(j,k)}}{dt} &= I_s^{(j-1,k)} - I_s^{(j,k)} - \mu^{(i,j,k)} I_s^{(j,k)} - \mu_s^{(j)} I_s^{(j,k)} + \delta_j \delta_s b_i b_{i,k} \sum_{j'} b^{(j')} F^{(j')} \\
&+ \sum_{k'=0}^{K-1} B_{k',k}^{(i,j)} I_s^{(j,k')} - \sum_{k'=0}^{K-1} B_{k,k'}^{(i,j)} I_s^{(j,k)} \\
&+ \delta_{s-1} \vartheta^{(i,j)} I_s^{(j,k)} - \delta_s \vartheta^{(i,j)} I_s^{(j,k)} \\
&+ \sum_{s'=2,s' \neq s}^{S} \delta_{\inf(s)-1} \gamma_{s',s}^{(i,j)} I_{s'}^{(j,k)} - \sum_{s'=2,s' \neq s}^{S} \delta_{\inf(s)-1} \gamma_{s,s'}^{(i,j)} I_s^{(j,k)} \\
&+ \delta_{\mathrm{sus}(s)-1} \lambda^{(i,j,k)} I_s^{(j,k)}
\end{aligned} \tag{6}$$

We will now break down the model equations line-by-line.

2.1 Line 1 - Aging, Births and Deaths

In line one, the first term represents the population aging into the current group. We assume that the population at $j = -1$ is identically zero for any remaining flags/states. The second term is the population leaving the current age group. The third term is the natural mortality rate multiplied by the population in the current group. The fourth term is the HIV/AIDS-related mortality rate multiplied by the population in the current group and current HIV state.

The last term is the birth term. Here we are using the notation that $\delta_p = 1$ if and only if $p = 0$ and is zero otherwise. Therefore the product of the four delta functions indicates that this term will only contribute to age group zero, and the sexually inactive population state $s = 0$. The summation gives the total births due to all age groups. Note that we are abusing notation: $F^{(j)}$ without subscripts indicates that all HIV states are to be summed over as well. This sum is then multiplied by the fraction $b_{i,k}$ which gives the fraction of the gender i birth rate that contributes to sexual activity group k, and b_i which gives the fraction of the birth rate that contributes to age group i. Remember that before sexual debut, the partner turnover rate in all sexual activity groups = 0.

2.2 Line 2 - Sexual Activity Group Migration

Line 2 represents the migration terms between activity groups. Recall that sexual activity is defined by sexual partner turnover rate, and age mixing preference. The first term accounts for the total population arriving in behaviour group k from all other groups k'. The second term accounts for the total population leaving group k for other groups k'.

2.3 Line 3 - Sexual Activity Debut

Line 3 represents the population change rates due to the current group becoming sexually active. It is assumed that the rates $\vartheta^{(i,j)}$ are zero below the earliest potential age of sexual debut. The first term adds to the $s = 1$ state (HIV uninfected but sexually active) from the inactive state $s = 0$. This same number is then removed from the $s = 0$ state in the second term.

2.4 Line 4 - Transitions between HIV Infected States

In this line we have the transitions between HIV infected states, including those states that include ART treatment. Here we use the function inf(s).

inf(s) = 1 if s >1, 0 otherwise

That is, inf(s) returns 1 provided state s is one of the HIV infected states; that is, if $s \neq 0$ and $s \neq 1$. Recall that, as always, $d_p = 1$ if and only if $p = 0$ and is zero otherwise. Therefore the first term adds to the current infected state while the second term subtracts from other infected states.

2.5 Line 5 - Force of Infection

This moves us from state $s = 1$ (HIV uninfected but sexually active) to state $s = 2$ (HIV infected stage 1).

2.6 Summary of Model Equations

Every term in the above equation is known (supplied by the user) except for the force of infection. This must be computed and we give the equations in the next section.

3 Force of Infection

In order to compute the force of infection we recall the following user input parameters:

- $\Delta_s^{(m,j,k)}$,$\Delta_s^{(f,j,k)}$ the desired partner turnover rates.
- $m_{j'}^{(m,j,k)}$, $m_{j'}^{(f,j,k)}$ the mixing relations i.e. the proportion of the total desired partners belonging to age group j'.
- $\phi_{s'}^{(m,j,k)}$,$\phi_{s'}^{(f,j,k)}$ the transmission probability of HIV per sex act. Although the model allows this probability to vary by all groups and states, in most of our work we will not vary this parameter by age. In some cases, we may vary this probability by sexual activity group, to be used as an indirect proxy for a higher likelihood of co-infection with other STDs among those in the higher sexual activity groups.
- $\psi^{(m,j,k)}$, $\psi^{(f,j,k)}$ the number of sex acts per partnership.

3.1 The base rate of infection

The base rate of infection of HIV **per partnership** can be computed using the probability of infection per sex act and the number of sex acts per partnership. The probability of not being infected in a single sex act is $(1-\phi_{s'}^{(i,j,k)})$ where i is either m or f for males or females. Then the probability of being infected in the total number of sex acts (assuming the partner is infected) is:

$$\beta_{s'}^{(i,j,k)} = 1-(1-\phi_{s'}^{(i,j,k)})^{\psi^{(i,j,k)}} \tag{7}$$

Here $\beta_{s'}^{(i,j,k)}$ represents the probability of getting infected per partner in HIV state s'. In our current modelling work, we fit our model using the probability of transmission per partnership as input, rather than the probability of transmission per sex act. We therefore simply set ψ to 1 for all groups and states, and consider ϕ as the probability of transmission per partnership. Therefore:

$$\beta_{s'}^{(i,j,k)} = 1-(1-\phi_{s'}^{(i,j,k)}) = \phi_{s'}^{(i,j,k)} \tag{8}$$

The probability of infection from any partners in HIV state s' is then equal to one minus the probability of not being infected by any partner in state s':

$$\lambda_{s'}^{(i,j,k)} = 1-(1-\beta_{s'}^{(i,j,k)})^{\Delta_{s,s'}^{(i,j,k)}} \tag{9}$$

Here, $\Delta_{s,s'}^{(i,j,k)}$ is the partner turnover rate with people infected and in HIV state s', which we will see later is derived from $\Delta_s^{(i,j,k)}$, the total desired partner turnover rate entered as input to the model. That is, $\Delta_s^{(i,j,k)}$ is the sum:

$$\Delta_s^{(i,j,k)} = \sum_{s'} \Delta_{s,s'}^{(i,j,k)} \tag{10}$$

where $\Delta_{s,s'}^{(i,j,k)}$ is the product of $\Delta_s^{(i,j,k)}$ and the proportion of the partnership pool with whom group (i,j,k) mixes, that is in HIV state s'.

The force of infection is then equal to one minus the probability of not being infected by any partner in any HIV state:

$$\lambda^{(i,j,k)} = 1 - \prod_{s'}(1 - \lambda_{s'}^{(i,j,k)}) \tag{11}$$

3.2 Determining the partner turnover rates per HIV state

To compute the force of infection we must compute $\Delta_{s,s'}^{(i,j,k)}$, the desired partner turnover rate with people in a given state. The number of available partners in a given group and state is:

$$P_s^{(i,j,k)} = I_s^{(j,k)} \Delta_s^{(i,j,k)}, \quad s > 0, \quad 0 \text{ otherwise} \tag{12}$$

Recall that HIV state 0 is used to represent the sexually inactive population and uninfected with HIV. It is assumed that the desired turnover rate for the sexually inactive state $s = 0$ is identically zero. The partner turnover rate with a given state s' is:

$$\Delta_{s,s'}^{(i,j,k)} = \sum_{j'=0}^{J-1} \sum_{k'=0}^{K-1} \Delta_s^{(i,j,k),(j',k')} \frac{P_{s'}^{(i',j',k')}}{\sum\limits_{s'} P_{s'}^{(i',j',k')}} \tag{13}$$

This equation is explained as follows. Denote the opposite gender as i'. The last term provides the ratio of partnerships on offer for a given state s' over all to the total available partners on offer in all HIV states. The newly introduced term $\Delta_s^{(i,j,k),(j',k')}$ is the number of partnerships desired with the primed group (yet to be computed). That is, it is a subset of $\Delta_s^{(i,j,k)}$: $\Delta_s^{(i,j,k),(j',k')} \subset \Delta_s^{(i,j,k)}$. Summing this over all age and sexual activity groups gives the total number of desired partners with a given state s'.

3.2.1 Determining the partner turnover rate with specific groups

We now determine $\Delta_s^{(i,j,k),(j',k')}$. This is computed based on age mixing preferences and sexual activity group mixing preferences.

Specifically, $\Delta_s^{(i,j,k),(j',k')}$ is computed as follows:

$$\Delta_s^{(i,j,k),(j',k')} = \Delta_s^{(i,j,k)} \left[(1 - \epsilon_k)\delta_{k,k'} + \epsilon_k \left(\frac{\sum\limits_{s'} P_{s'}^{(i',j',k')}}{\sum\limits_{k''=0}^{K-1} \sum\limits_{s'} P_{s'}^{(i',j',k'')}} \right) \right] m_{j'}^{(i,j,k)} \tag{14}$$

The first multiplicative term in the above equation is the overall desired turnover rate for the population with HIV state s, gender i, age group j, and sexual activity group k, provided as user input. Next we have the **activity group assortativity parameter** ϵ_k which ranges between 0 and 1. In the case that ϵ_k is zero, the first large multiplicative term tells us

that there is no cross-mixing by sexual activity group; this would represent 100% assortative mixing by activity group. In the case that $\epsilon_k = 1$, the first term is zero and the first multiplication is by the ratio of the total partnerships on offer in group k' to the total partnerships on offer over all activity groups k''. Therefore multiplying this ratio by the desired partnership turnover rate $\Delta_s^{(i,j,k)}$ would distribute these partnerships according to the available ratio of partnerships on offer from the partner group. This would represent 100% random mixing by activity group, based on partnerships available. Any value of ϵ_k greater than 0 and less than 1 would represent partially assortative and partially random mixing.

We then multiply by $m_{j'}^{(i,j,k)}$, that is the mixing factor with age group j' in order to adjust the desired partnerships by the fraction of desired partners in age group j'.

3.3 Partnership Turnover Balancing

Most often, the total number of partnerships requested from j', k' for j, k will not match exactly the total number of partnerships requested from j, k for j', k'. That is, most often:

$$\sum_{s=1}^{S} I_s^{(j,k)} \Delta_s^{(i,j,k),(j',k')} \neq \sum_{s'=1}^{S} I_{s'}^{\prime(j',k')} \Delta_{s'}^{(i',j',k'),(j,k)} \tag{15}$$

We now discuss balancing partner turnover rates to match partnerships available. This is done as follows:

$$BL^{(j,k),(j',k')} = \frac{\sum\limits_{s=1} F_s^{(j,k)} \Delta_s^{(f,j,k),(j',k')}}{\sum\limits_{s=1} M_s^{(j',k')} \Delta_s^{(m,j',k'),(j,k)}} \tag{16}$$

This equation is explained as follows: The sum of F in the numerator over s gives the total active female population (note that $s = 0$ is not included in the sum). Multiplying this population by by the rate $\Delta_s^{(f,j,k),(j',k')}$ gives the total number of partners that this population desires with men in the primed group.

The denominator is analogous except for men and gives the total number of partners that men in the primed group want with women in the unprimed group.

If the ratio of these two quantities is more than unity, women in the index (unprimed) group desire more partners per time than men in the target (primed) group. If the ratio is less than unity, men in in the target group desire more partners than women in the index group.

We then adjust the partner turnover rates as follows:

$$\begin{aligned}\Delta A_s^{(m,j,k),(j',k')} &= \Delta_s^{(m,j,k),(j',k')} \times \left(BL^{(j,k),(j',k')}\right)^{\theta} \\ \Delta A_s^{(f,j,k),(j',k')} &= \Delta_s^{(f,j,k),(j',k')} \times \left(BL^{(j,k),(j',k')}\right)^{-(1-\theta)}\end{aligned} \tag{17}$$

Where θ is a compromise parameter between 0 and 1. If θ is zero, men get what they want, i.e. they get their desired rates. In this case, if BL is greater than unity (women desire more than men), the rate for women will be reduced by a ratio of BL. If θ is 1, women get what they want. If BL is greater than 1, this means that the rate for men will be increased by a factor of BL.

3.4 Turnover rate equation flow

Once the adjusted partner turnover rates are computed in eq (17), these adjusted rates are used to compute the turnover rates with a given state s in eq (13). From the state turnover rates we compute the HIV infected turnover rates in eq (10) and finally the force of infection in eq (11).

Table S1. Model Input Ranges (note: in all subscripts, i=gender, j=age, k=sexual activity group)

Model Input Parameter (Symbol used in equations)	**Range**	**Source**	**Value in top fitting scenario**
Demography			
Fertility rates per year (b^j)	Fixed (non-varying)	Analysis of Masaka (Uganda) cohort data	Age 15-19: 0.180 35-39: 0.211 20-29: 0.333 40-44: 0.079 30-34: 0.275 45-49: 0.028
Non HIV-related mortality rates per year (μ_i^j)	Fixed (non-varying):	Cohort data analysis, fitted to Brass life table model[1]	Life expectancy Females: 69 yrs Males: 66 yrs
Biology			
HIV-related mortality rate (μ_s), where s=one of the HIV (not yet AID) HIV states	0.04-0.10/year	Cohort anlysis	0.065/year
AIDS-related mortality rate (μ_s), where s = AIDS state	0.7-1.3/year (~ 0.04-0.11/month)	[2]	1.29/year
HIV infection probability per partnership (ϕ_s^i) Infection probabilities for HIV stage 1, 3, 4 are multiples of Stage 2, to ensure infectiousness from highest to lowest in stages 1, 4, 3, 2, respectively. (Stage 4 = AIDS)	HIV Stage 2: M to F: 0.01-0.08 F to M: (male prob) * (0.05-0.90) M to F by stage: Stage 1: 0.05-0.72 Stage 3: 0.0125-0.54 Stage 4: 0.0188-0.71	[3-5]	M->F: 6.7% F->M: 5.5% M->F: S1: 29% S3: 19% S4: 28% F->M: S1: 21% S3: 14% S4: 21%
Mean duration in each HIV stage = 1/(rate of movement between HIV state s and s') ($\gamma_{s,s'}$)	Fixed (non-varying)	[2, 6]	Stage 1: 5 months, stage 2: 7 years, stage 3: 1.5 years, stage 4: 10 months
Year of first HIV infection	1970-1980	Earlier than 1982 [7]	1976
Sexual Behavior			
Earliest potential age at sexual debut	11-15	Cohort data analysis	14
Rate of becoming sexually active ($\vartheta^{i,j}$)	Beginning with earliest potential age at sexual debut, the rate of becoming sexually active first increases with age, and then decreases. Our data show that those not sexually active by a certain age have a very low rate of becoming sexually active.	Cohort data analysis	Age Sex Rate/year 14-15 M 0.032 F 0. 023 16-19 M 0.128 F 0.138 20-24 M 0.214 F 0.253 25-28 M 0.160 F 0.184 29+ M 0.096 F 0.161
Desired partner turnover rate by gender, age, sexual activity group, not in AIDS state ($\Delta^{i,j,k}$)	Until age 30, the ranges for ($\Delta_{i,j,k}$): High activity: 20-90 Middle activity: 1-5	Cohort data analysis	$\Delta_{i,j,k}$ for Age < 30: Sex Group Rate/yr

	Low activity: (middle activity rate) * (0.1 – 0.5) Age 30+: $\Delta_{i,j,k}$ reduces by 3%/year		M High 68.24 Mid 1.99 Low 0.25 F High 36.62 Mid 2.96 Low 1.15
Desired partner turnover rate by gender, age, sexual activity group, in AIDS state ($\Delta^{i,j,k}$)	($\Delta^{i,j,k}$) * (0.20 – 0.80)	Cohort data analysis (with limited data)	0.41
Proportion entering each activity group at sexual debut - based on partner turnover desire (b_k)	High: 0.1%-3.0% Middle: 30% - 85% Low: 100%-high-middle	Cohort data analysis	High: 2.7% Middle: 62.0% Low: 35.3%
Cross generational age mixing: Proportion entering each 'cross generational' activity group – based on desired age mixing: (b_k) * (proportion of women willing to have cross generational partners)	Range at sexual debut: 0-50% Rate of leaving cross-generational age mixing group: 0.00-0.10	Cohort data analysis	31% at sexual debut. Starting at age 25, the rate of leaving this compartment (among women) = 0.060/year.
Age mixing (non-x-generational) Applies to partnerships that are not with women willing to engage in cross-generational sex.	Maximum age difference between partners: 2-7 years. 80% of partnerships are desired with older males, while 20% with younger/same age males.	Cohort data analysis	± 3 years
Assortativity of mixing by sexual activity group (ε_k)	0 to 1 0 = assortative 1 = proportional to partnerships on offer	[8]	0.66
Balancing parameter (θ)	0 to 1 0 = men get what they are requesting and women increase/decrease partnerships to match male demand. 1 = women get what they request. Between 0-1 = a compromise.	[9]	0.98
Year of intentional behavior change	1st year of change: 1988-1991 2nd year of change: 1998-2000	Cohort data analysis, and [10]	1988.7 1998.0
Factor changing desired sexual partner turnover rates: New $\Delta_{i,j,k}$ = (Old $\Delta_{i,j,k}$)*factor	1988-1991: 0.44 – 0.99 1998-2000: 1.01 – 2.01	Turnover rates change gradually over 2 years.	1988.7: 0.97 1998.0: 1.02

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